

§ 2.5

6 – 44 (even)

5 – 18 A quadratic Function is Given.

(a) Express the quadratic function in standard form.

(b) Find its vertex and its x - and y -intercept(s).

(c) Sketch its graph.

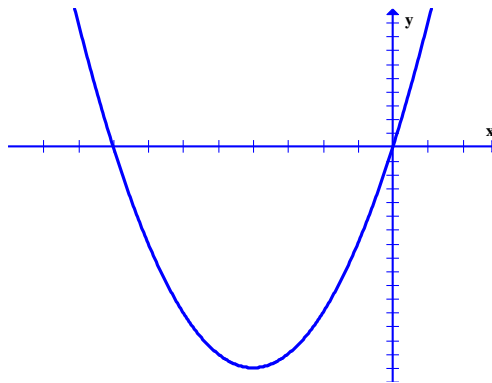
6) $f(x) = x^2 + 8x$

$$\begin{aligned} f(x) &= x^2 + 8x \\ &= (x^2 + 8x + 16) - 16 \\ &= (x + 4)^2 - 16 \end{aligned}$$

The vertex is $(-4, -16)$.

y -intercept: 0

x -intercepts: 0, -8



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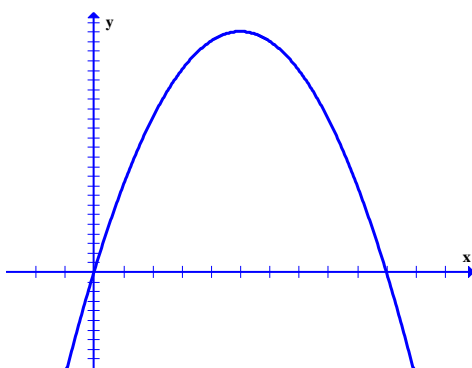
8) $f(x) = -x^2 + 10x$

$$\begin{aligned} f(x) &= -x^2 + 10x \\ &= -(x^2 - 10x + 25) + 25 \\ &= -(x - 5)^2 + 25 \end{aligned}$$

The vertex is (5,25).

y-intercept: 0

x-intercepts: 0, 10



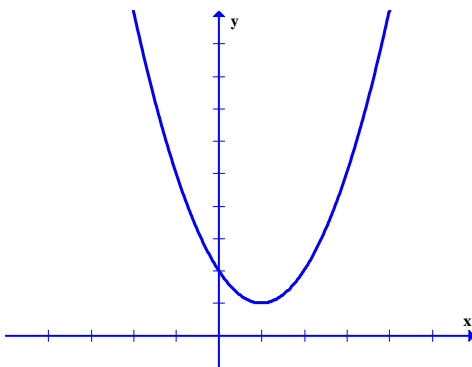
10) $f(x) = x^2 - 2x + 2$

$$\begin{aligned} f(x) &= x^2 - 2x + 2 \\ &= (x^2 - 2x + 1) + 2 - 1 \\ &= (x - 1)^2 + 1 \end{aligned}$$

The vertex is (1,1).

y-intercept: 2

x-intercepts: none



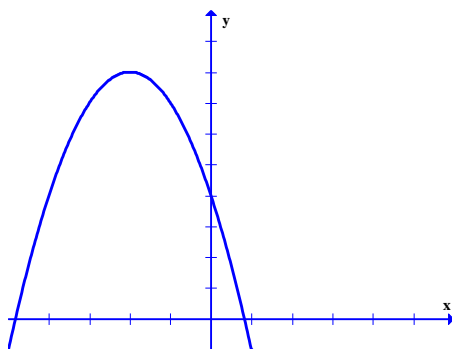
$$12) f(x) = -x^2 - 4x + 4$$

$$\begin{aligned} f(x) &= -x^2 - 4x + 4 \\ &= -(x^2 + 4x + 4) + 4 + 4 \\ &= -(x + 2)^2 + 8 \end{aligned}$$

The vertex is $(-2, 8)$.

y -intercept: 4

x -intercepts: $-2 + 2\sqrt{2}$ and $-2 - 2\sqrt{2}$



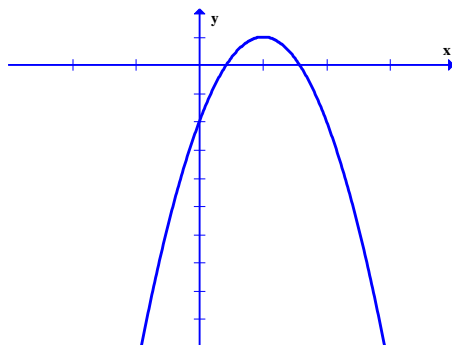
$$14) f(x) = -3x^2 + 6x - 2$$

$$\begin{aligned} f(x) &= -3x^2 + 6x - 2 \\ &= -3(x^2 - 2x + 1) - 2 + 3 \\ &= -3(x - 1)^2 + 1 \end{aligned}$$

The vertex is $(1, 1)$.

y -intercept: -2

x -intercepts: $1 + \frac{\sqrt{3}}{3}$ and $1 - \frac{\sqrt{3}}{3}$



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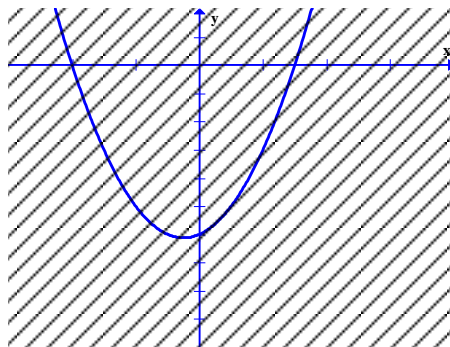
16) $f(x) = 2x^2 + x - 6$

$$\begin{aligned} f(x) &= 2x^2 + x - 6 \\ &= 2\left(x^2 + \frac{1}{2}x + \frac{1}{16}\right) - 6 - \frac{1}{8} \\ &= 2\left(x + \frac{1}{4}\right)^2 - \frac{49}{8} \end{aligned}$$

The vertex is $\left(-\frac{1}{4}, -\frac{49}{8}\right)$

y -intercept: -6

x -intercepts: $\frac{3}{2}$ and -2



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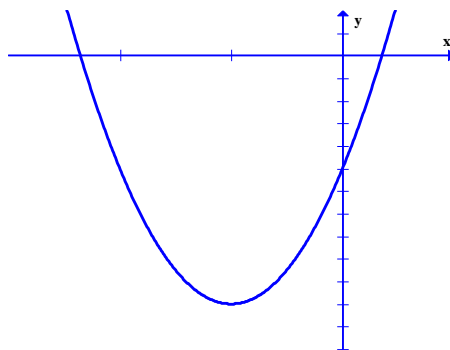
18) $f(x) = 6x^2 + 12x - 5$

$$\begin{aligned} f(x) &= 6x^2 + 12x - 5 \\ &= 6(x^2 + 2x + 1) - 5 - 6 \\ &= 6(x + 1)^2 - 11 \end{aligned}$$

The vertex is $(-1, -11)$

y -intercept: -5

x -intercepts: $-1 + \sqrt{\frac{11}{6}}$ and $-1 - \sqrt{\frac{11}{6}}$



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19 – 28 A quadratic function is given.

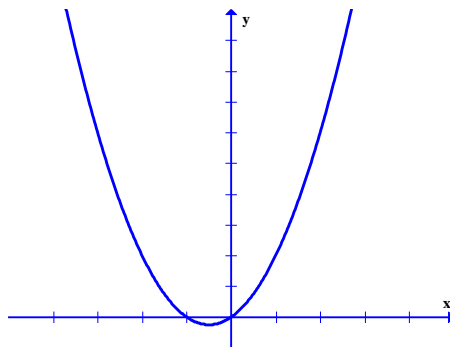
(a) Express the quadratic function in standard form.

(b) Sketch its graph.

(c) Find its maximum or minimum value.

20) $f(x) = x + x^2$

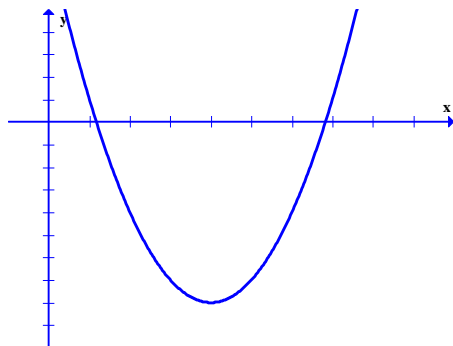
$$\begin{aligned} f(x) &= x^2 + x \\ &= \left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} \\ &= \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} \end{aligned}$$



minimum value: $-\frac{1}{4}$

22) $f(x) = x^2 - 8x + 8$

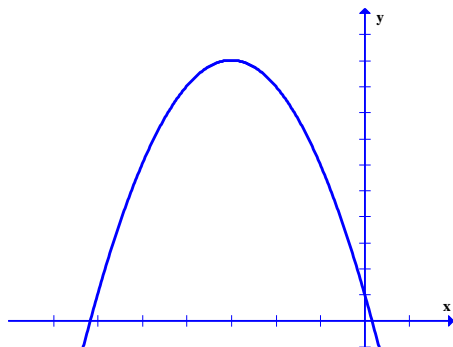
$$\begin{aligned} f(x) &= x^2 - 8x + 8 \\ &= (x^2 - 8x + 16) + 8 - 16 \\ &= (x - 4)^2 - 8 \end{aligned}$$



minimum value: -8

24) $f(x) = 1 - 6x - x^2$

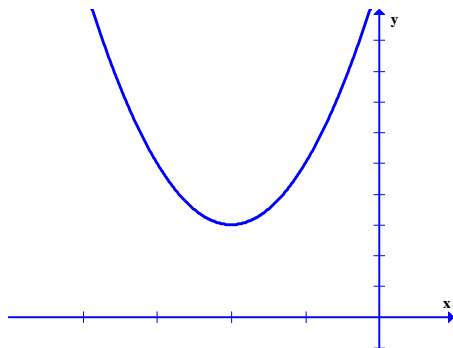
$$\begin{aligned} f(x) &= -x^2 - 6x + 1 \\ &= -(x^2 + 6x + 9) + 1 + 9 \\ &= -(x + 3)^2 + 10 \end{aligned}$$



maximum value: 10

26) $f(x) = 2x^2 + 8x + 11$

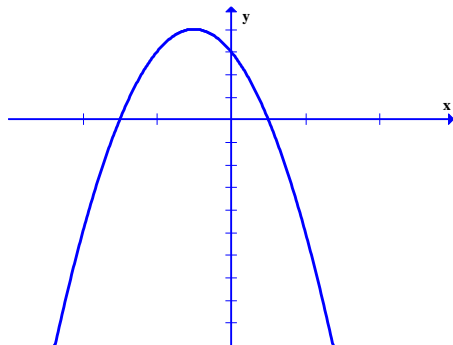
$$\begin{aligned} f(x) &= 2x^2 + 8x + 11 \\ &= 2(x^2 + 4x + 4) + 11 - 8 \\ &= (x + 2)^2 + 3 \end{aligned}$$



minimum value: 3

28) $f(x) = 3 - 4x - 4x^2$

$$\begin{aligned} f(x) &= -4x^2 - 4x + 3 \\ &= -4\left(x^2 + x + \frac{1}{4}\right) + 3 + 1 \\ &= \left(x + \frac{1}{2}\right)^2 + 4 \end{aligned}$$



maximum value: 4

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29 – 38 Find the maximum or minimum value of the function

$$30) f(x) = 1 + 3x - x^2$$

$$a = -1, b = 3, \text{ and } c = 1$$

Since $a < 0$, f has a maximum value at

$$\begin{aligned} c - \frac{b^2}{4a} &= 1 - \frac{3^2}{4(-1)} \\ &= 1 + \frac{9}{4} \\ &= \frac{13}{4} \end{aligned}$$

$$32) f(t) = 10t^2 + 40t + 113$$

$$a = 10, b = 40, \text{ and } c = 113$$

Since $a > 0$, f has a minimum value at

$$\begin{aligned} c - \frac{b^2}{4a} &= 113 - \frac{40^2}{4(10)} \\ &= 113 - 40 \\ &= 73 \end{aligned}$$

$$34) f(x) = 100x^2 - 1500x$$

$$a = 100, b = 1500, \text{ and } c = 0$$

Since $a > 0$, f has a minimum value at

$$\begin{aligned} c - \frac{b^2}{4a} &= 0 - \frac{(1500)^2}{4(100)} \\ &= -\frac{2250000}{400} \\ &= -5625 \end{aligned}$$

$$36) f(x) = -\frac{x^2}{3} + 2x + 7$$

$$a = -\frac{1}{3}, b = 2, \text{ and } c = 7$$

Since $a < 0$, f has a maximum value at

$$\begin{aligned} c - \frac{b^2}{4a} &= 7 - \frac{2^2}{4\left(-\frac{1}{3}\right)} \\ &= 7 + 3 \\ &= 10 \end{aligned}$$

$$38) f(x) = 2x(x - 4) + 7 = 2x^2 - 8x + 7$$

$$a = 2, b = -8, \text{ and } c = 7$$

Since $a > 0$, f has a minimum value at

$$\begin{aligned} c - \frac{b^2}{4a} &= 7 - \frac{(-8)^2}{4(2)} \\ &= 7 - 8 \\ &= -1 \end{aligned}$$

40) Find a function whose graph is a parabola with vertex $(3,4)$ and that passes through the point $(1,-8)$

Since the graph of f is a parabola, we know that $f(x) = a(x - h)^2 + k$. Since the vertex is at the point $(3,4)$, we know that $h = 3$ and $k = 4$. Thus

$$f(x) = a(x - 3)^2 + 4.$$

Now, we just need to find the value of a .

Since the graph passes through the point $(1,-8)$, we know that

$$-8 = f(1) = a(1 - 3)^2 + 4 = 4a + 4.$$

Solving for a we get $a = -3$, and therefore $f(x) = -3(x - 3)^2 + 4$.

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41 – 44 Find the domain and range of the function.

$$42) f(x) = x^2 - 2x - 3$$

The domain is the set of all real number.

Since $a = 1, b = -2, c = -3$, and $a > 0$, we get that f has a minimum value of

$$\begin{aligned} c - \frac{b^2}{4a} &= -3 - \frac{(-2)^2}{4(1)} \\ &= -3 - 1 \\ &= -4 \end{aligned}$$

Thus the range of f is $(-4, \infty)$.

$$44) f(x) = -3x^2 + 6x + 4$$

The domain is the set of all real number.

Since $a = -3, b = 6, c = 4$, and $a < 0$, we get that f has a maximum value of

$$\begin{aligned} c - \frac{b^2}{4a} &= 4 - \frac{6^2}{4(-3)} \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

Thus the range of f is $(-\infty, 7)$.